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Correlated squeezed states of two coupled oscillators with delta-kicked frequencies

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Abstract

The squeezing phenomenon due to parametric excitations in a system of two coupled harmonic oscillators with delta-kicked frequencies is investigated. The covariance matrix of the correlated squeezed states generated in this system is calculated explicitly using the phase-space approach.

1. Introduction

The aim of this work is to study the squeezing phenomenon in a system of coupled harmonic oscillators due to the parametric excitations. Our model is a system of two coupled harmonic oscillators with delta-kicked frequencies, which is a two-mode generalisation of the one-mode parametric oscillator considered in Refs. [1–3]. The advantage of this model is that its dynamics can be studied explicitly due to the validity of using the normal mode transformation and the exact solvability of the equations of motion.

The excitation of the correlated squeezed states [4] by the frequency time-dependence has been discussed for quantum oscillators in Refs. [5–9] and for oscillator chains in Refs. [10–12]. The squeezing phenomenon in a system with two coupled time-dependent oscillators has been studied in Refs. [8,13]. In this paper we apply the phase-space approach to this problem and derive the explicit covariance matrix of the correlated squeezed state generated in the system.

In the phase-space approach, quantum states are represented by the Winger functions [14,15]. Since the Hamiltonian of our model is quadratic in positions and momenta, the equation of motion of the Wigner function is of the same form as the classical Liouville equation, and the time evolution of the Wigner function will exactly follow the phase flow, i.e., the solution of the corresponding classical equation of motion. Therefore the phase flow completely determines the quantum dynamics of our model.

In Section 2, we solve the classical equations of motion of our model. In Section 3, we calculate the explicit covariance matrix of the correlated squeezed state excited out of a coherent state. In Sections 4 and 5, we discuss the applications and possible generalizations of our results.

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2. Coupled quantum oscillators with delta-kicked frequencies

Consider a system of two coupled oscillators with delta-kicking frequencies. The Hamiltonian of this system is defined as

$$\hat{H}=\tfrac{1}{2}(\hat{p}_1^2+\hat{p}_2^2)+\tfrac{1}{2}[\omega_0^2+k\delta(t)](\hat{q}_1^2+\hat{q}_2^2)+\tfrac{1}{2}[\lambda_0^2+\lambda\delta(t)](\hat{q}_1-\hat{q}_2)^2\,, \tag{1}$$

where $\delta(t)$ is the delta function, $\hat{q}_1$ and $\hat{q}_2$ are the position operators and $\hat{p}_1$ and $\hat{p}_2$ are the momentum operators for the first and second oscillator, respectively. The masses of these two squeezed oscillators are both set equal to unity, and $\hbar=1$ throughout this paper.

For the Hamiltonian defined above, the equations of motion of the mean values of the operators with respect to a given quantum state, i.e., the corresponding classical equations of motion, are as follows,

$$\ddot{q}_1+[\omega_0^2+k\delta(t)]q_1+[\lambda_0^2+\lambda\delta(t)](q_1-q_2)=0\,, \tag{2}$$

$$\ddot{q}_2+[\omega_0^2+k\delta(t)]q_2-[\lambda_0^2+\lambda\delta(t)](q_1-q_2)=0\,, \tag{3}$$

$$p_1=\dot{q}_1\,,\qquad p_2=\dot{q}_2\,, \tag{4}$$

where q_1, q_2, p_1 and p_2 are the mean values of the operators $\hat{q}_1$, $\hat{q}_2$, $\hat{p}_1$ and $\hat{p}_2$ with respect to the given state, respectively.

Introducing the orthogonal transformation on the coordinates q_1 and q_2,

$$\begin{pmatrix}x\\y\end{pmatrix}=\frac{1}{\sqrt{2}}\begin{pmatrix}1&-1\\1&1\end{pmatrix}\begin{pmatrix}q_1\\q_2\end{pmatrix}, \tag{5}$$

then the canonical transformation on the coordinates and momenta corresponding to this coordinate transformation takes the form

$$\begin{pmatrix}x\\p_x\\y\\p_y\end{pmatrix}=\frac{1}{\sqrt{2}}\begin{pmatrix}1&0&-1&0\\0&1&0&-1\\1&0&1&0\\0&1&0&1\end{pmatrix}\begin{pmatrix}q_1\\p_1\\q_2\\p_2\end{pmatrix}\equiv M\begin{pmatrix}q_1\\p_1\\q_2\\p_2\end{pmatrix}. \tag{6}$$

Using these new canonical variables, the equations of motion (2)–(4) become

$$\ddot{x}+[\omega_x^2+(k+2\lambda)\delta(t)]x=0\,,\qquad \omega_x=\sqrt{\omega_0^2+2\lambda_0^2}\,, \tag{7}$$

$$\ddot{y}+[\omega_y+k\delta(t)]y=0\,,\qquad \omega_y=\omega_0\,, \tag{8}$$

$$p_x=\dot{x}\,,\qquad p_y=\dot{y}\,. \tag{9}$$

When $t<0$ or $t>0$, since $\delta(t)=0$ in these time intervals, the solutions for (7)–(9) are rotations in the phase spaces (x,p_x) and (y,p_y),

$$\begin{pmatrix}x(t)\\p_x(t)\end{pmatrix}=\begin{pmatrix}\cos(\omega_x\tau)&\omega_x^{-1}\sin(\omega_x\tau)\\-\omega_x\sin(\omega_x\tau)&\cos(\omega_x\tau)\end{pmatrix}\begin{pmatrix}x(t_0)\\p_x(t_0)\end{pmatrix}, \tag{10}$$

$$\begin{pmatrix}y(t)\\p_y(t)\end{pmatrix}=\begin{pmatrix}\cos(\omega_y\tau)&\omega_y^{-1}\sin(\omega_y\tau)\\-\omega_y\sin(\omega_y\tau)&\cos(\omega_y\tau)\end{pmatrix}\begin{pmatrix}y(t_0)\\p_y(t_0)\end{pmatrix}, \tag{11}$$

where $t_0<t<0$ or $t>t_0>0$, and $\tau=t-t_0>0$.

In terms of the original variables, Eqs. (10) and (11) take the form

$$\begin{pmatrix} q_1(t) \\ p_1(t) \\ q_2(t) \\ p_2(t) \end{pmatrix} = R(t, t_0) \begin{pmatrix} q_1(t_0) \\ p_1(t_0) \\ q_2(t_0) \\ p_2(t_0) \end{pmatrix}, \tag{12}$$

with (T denotes the transpose of a matrix)

$$R(t, t_0) = M^{\mathrm{T}} \begin{pmatrix} \cos(\omega_x \tau) & \omega_x^{-1} \sin(\omega_x \tau) & 0 & 0 \\ -\omega_x \sin(\omega_x \tau) & \cos(\omega_x \tau) & 0 & 0 \\ 0 & 0 & \cos(\omega_y \tau) & \omega_y^{-1} \sin(\omega_y \tau) \\ 0 & 0 & -\omega_y \sin(\omega_y \tau) & \cos(\omega_y \tau) \end{pmatrix} M. \tag{13}$$

Across $t=0$, Eqs. (7)–(9) are still exactly integrable, and the solutions are the following shearing transformations,

$$\begin{pmatrix} x(0^+) \\ p_x(0^+) \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ -\mu & 1 \end{pmatrix} \begin{pmatrix} x(0^-) \\ p_x(0^-) \end{pmatrix}, \qquad \mu \equiv k + 2\lambda, \tag{14}$$

$$\begin{pmatrix} y(0^+) \\ p_y(0^+) \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ -k & 1 \end{pmatrix} \begin{pmatrix} y(0^-) \\ y_x(0^-) \end{pmatrix}. \tag{15}$$

Correspondingly, in terms of the original variables

$$\begin{pmatrix} q_1(0^+) \\ p_1(0^+) \\ q_2(0^+) \\ p_2(0^+) \end{pmatrix} = S \begin{pmatrix} q_1(0^-) \\ p_1(0^-) \\ q_2(0^-) \\ p_2(0^-) \end{pmatrix}, \tag{16}$$

where

$$S = M^{\mathrm{T}} \begin{pmatrix} 1 & 0 & 0 & 0 \\ -\mu & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & -k & 1 \end{pmatrix} M. \tag{17}$$

The phase flow, i.e., the general solution of the classical equations of motion, of this system will be denoted as

$$\begin{pmatrix} q_1(t) \\ p_1(t) \\ q_2(t) \\ p_2(t) \end{pmatrix} = U(t, t_0) \begin{pmatrix} q_1(t_0) \\ p_1(t_0) \\ q_2(t_0) \\ p_2(t_0) \end{pmatrix}, \tag{18}$$

where $t, t_0 \approx \mathbb{R}$. This expression contains (12) and (16) as two special cases.

3. Time evolution of the quantum states

The covariance matrix C of a given quantum state is defined as a 4×4 symmetric matrix of the form

$$C = \begin{pmatrix} \sigma_{q_1 q_1} & \sigma_{q_1 p_1} & \sigma_{q_1 q_2} & \sigma_{q_1 p_2} \\ \sigma_{q_1 p_1} & \sigma_{p_1 p_1} & \sigma_{q_2 p_1} & \sigma_{p_1 p_2} \\ \sigma_{q_1 q_2} & \sigma_{q_2 p_1} & \sigma_{q_2 q_2} & \sigma_{q_2 p_2} \\ \sigma_{q_1 p_2} & \sigma_{p_1 p_2} & \sigma_{q_2 p_2} & \sigma_{p_2 p_2} \end{pmatrix}, \tag{19}$$

where $(i, j=1, 2)$

$$\sigma_{q_iq_j} = \langle \hat{q}_i\hat{q}_j \rangle - \langle \hat{q}_i \rangle \langle \hat{q}_j \rangle \,, \tag{20}$$

$$\sigma_{p_ip_j} = \langle \hat{p}_i\hat{p}_j \rangle - \langle \hat{p}_i \rangle \langle \hat{p}_j \rangle \,, \tag{21}$$

$$\sigma_{q_ip_j} = \langle \tfrac{1}{2}(\hat{q}_i\hat{p}_j + \hat{p}_j\hat{q}_i) \rangle - \langle \hat{q}_i \rangle \langle \hat{p}_j \rangle \,. \tag{22}$$

Corresponding to (18), the time evolution of the covariance matrix is

$$C(t) = U(t, t_0)C(t_0)[U(t, t_0)]^{\mathrm{T}} \,. \tag{23}$$

Note that it is independent of the time evolution of the mean values.

Since the Wigner function of a correlated squeezed coherent state is in general a Gaussian distribution, it is completely determined by the mean values and the covariance matrix of the corresponding state. Therefore the time evolution of the Wigner function, hence that of the correlated squeezed coherent state, is determined by (18) and (23). Because the mean values correspond to semiclassical behavior, we shall only concentrate on the evolution of the covariance matrix in this paper.

Assume at $t<0$, the quantum state in our model to be a coherent state (with the ground state as its special case). The covariance matrix of this state is then constant before the delta-kicks,

$$C(t<0) = M^{\mathrm{T}} C_{xy}(t<0) M \,, \tag{24}$$

where

$$C_{xy}(t<0) = \frac{1}{2} \begin{pmatrix} \omega_x^{-1} & 0 & 0 & 0 \\ 0 & \omega_x & 0 & 0 \\ 0 & 0 & \omega_y^{-1} & 0 \\ 0 & 0 & 0 & \omega_y \end{pmatrix} \tag{25}$$

is the covariance matrix of the coherent state in the basis of the new variables.

For $t>0$, according to (23) the covariance matrix becomes

$$C(t>0) = R(t, 0) S C(t<0) S^{\mathrm{T}} [R(t, 0)]^{\mathrm{T}} \,, \tag{26}$$

where $R(t, 0)$ and S are defined by (13) and (17), respectively.

3.1. Coordinate dispersions

Using (26), it is easy to calculate the following coordinate dispersions for $t>0$,

$$\sigma_{q_1q_1}(t>0) = \sigma_{q_2q_2}(t>0) = \frac{1}{4}\left(\frac{1}{\omega_x} + \frac{1}{\omega_y}\right) - \frac{1}{4}\left(\frac{\mu \sin(2\omega_x t)}{\omega_x^2} + \frac{k \sin(2\omega_y t)}{\omega_y^2}\right)$$
$$+ \frac{1}{4}\left(\frac{\mu^2 \sin^2(\omega_x t)}{\omega_x^3} + \frac{k^2 \sin^2(\omega_y t)}{\omega_y^3}\right), \tag{27}$$

$$\sigma_{q_1q_2}(t>0) = \frac{1}{4}\left(-\frac{1}{\omega_x} + \frac{1}{\omega_y}\right) + \frac{1}{4}\left(\frac{\mu \sin(2\omega_x t)}{\omega_x^2} - \frac{k \sin(2\omega_y t)}{\omega_y^2}\right)$$
$$+ \frac{1}{4}\left(-\frac{\mu^2 \sin^2(\omega_x t)}{\omega_x^3} + \frac{k^2 \sin^2(\omega_y t)}{\omega_y^3}\right). \tag{28}$$

According to (24), the corresponding dispersions for $t<0$ are

$$\sigma_{q_1q_1}(t<0)=\sigma_{q_2q_2}(t<0)=\frac{1}{4}\left(\frac{1}{\omega_x}+\frac{1}{\omega_y}\right), \tag{29}$$

$$\sigma_{q_1q_2}(t<0)=\frac{1}{4}\left(-\frac{1}{\omega_x}+\frac{1}{\omega_y}\right). \tag{30}$$

When the interaction between the two oscillators appears only in a very short period of time, it is equivalent to the case that λ_0 in Hamiltonian (1) vanishes and $\omega_x=\omega_y=\omega_0$. Under this condition, we have

$$\sigma_{q_1q_1}(t>0)=\sigma_{q_2q_2}(t>0)=\frac{1}{2\omega_0}-\frac{(\lambda+k)\sin(2\omega_x t)}{2\omega_0^2}+\frac{(k^2+2\lambda^2+2k\lambda)\sin^2(\omega_x t)}{2\omega_0^3}, \tag{31}$$

$$\sigma_{q_1q_2}(t>0)=\frac{\lambda\sin(2\omega_0 t)}{2\omega_0^2}-\frac{\lambda(\lambda+k)\sin^2(\omega_x t)}{\omega_x^3}, \tag{32}$$

and

$$\sigma_{q_1q_1}(t<0)=\sigma_{q_2q_2}(t<0)=\frac{1}{2\omega_0}, \tag{33}$$

$$\sigma_{q_1q_2}(t<0)=0. \tag{34}$$

3.2. Momentum dispersions

Analogous to the coordinate dispersions, the momentum dispersions can also be obtained from (26) as follows,

$$\sigma_{p_1p_1}(t>0)=\sigma_{p_2p_2}(t>0)=\tfrac{1}{4}(\omega_x+\omega_y)+\tfrac{1}{4}[\mu\sin(2\omega_x t)+k\sin(2\omega_y t)]$$
$$+\frac{1}{4}\left(\frac{\mu^2\cos^2(\omega_x t)}{\omega_x}+\frac{k^2\cos^2(\omega_y t)}{\omega_y}\right), \tag{35}$$

$$\sigma_{p_1p_2}(t>0)=\tfrac{1}{4}(-\omega_x+\omega_y)+\tfrac{1}{4}[-\mu\sin(2\omega_x t)+k\sin(2\omega_y t)]$$
$$+\frac{1}{4}\left(-\frac{\mu^2\cos^2(\omega_x t)}{\omega_x}+\frac{k^2\cos^2(\omega_y t)}{\omega_y}\right), \tag{36}$$

and the corresponding dispersions for $t<0$ are

$$\sigma_{p_1p_1}(t>0)=\sigma_{p_2p_2}(t>0)=\tfrac{1}{4}(\omega_x+\omega_y), \tag{37}$$

$$\sigma_{p_1p_2}(t<0)=\tfrac{1}{4}(-\omega_x+\omega_y). \tag{38}$$

3.3. Correlations

From (26), we can also obtain the time evolution of the correlations as follows,

$$\sigma_{q_1p_1}(t>0)=\sigma_{q_2p_2}(t>0)=-\frac{1}{4}\left(\frac{\mu\cos(2\omega_x t)}{\omega_x}+\frac{k\cos(2\omega_y t)}{\omega_y}\right)+\frac{1}{8}\left(\frac{\mu^2\sin(2\omega_x t)}{\omega_x^2}+\frac{k^2\sin(2\omega_y t)}{\omega_y^2}\right), \tag{39}$$

$$\sigma_{q_1p_2}(t>0)=\sigma_{q_2p_1}(t>0)=\frac{1}{4}\left(\frac{\mu\cos(2\omega_x t)}{\omega_x}-\frac{k\cos(2\omega_y t)}{\omega_y}\right)+\frac{1}{8}\left(-\frac{\mu^2\sin(2\omega_x t)}{\omega_x^2}+\frac{k^2\sin(2\omega_y t)}{\omega_y^2}\right), \tag{40}$$

and correspondingly for $t<0$

$$\sigma_{q_1p_1}(t<0)=\sigma_{q_2p_2}(t<0)=\sigma_{q_1p_2}(t<0)=\sigma_{q_2p_1}(t<0)=0\,. \tag{41}$$

3.4. Squeezing parameters and correlation coefficients

The squeezing coefficients K_i ($i=1, 2$) are defined according to the following relation,

$$K_i^2(t>0)=\frac{\sigma_{q_iq_i}(t>0)}{\sigma_{q_iq_i}(t<0)}\,. \tag{42}$$

Upon substitution of (27) and (29) into the above definition, we get

$$\begin{aligned}K_i^2(t>0)&=1-\frac{1}{(\omega_x+\omega_y)}\left(\frac{\mu\omega_y\sin(2\omega_xt)}{\omega_x}+\frac{k\omega_x\sin(2\omega_yt)}{\omega_y}-\frac{\mu^2\omega_y\sin^2(\omega_xt)}{\omega_x^2}-\frac{k^2\omega_x\sin^2(\omega_yt)}{\omega_y^2}\right)\\&=1+\frac{\mu^2\omega_y}{2\omega_x^2(\omega_x+\omega_y)}+\frac{k^2\omega_x}{2\omega_y^2(\omega_x+\omega_y)}-\frac{\mu\omega_y}{\omega_x(\omega_x+\omega_y)}\left(1+\frac{\mu^2}{4\omega_x^2}\right)^{1/2}\sin(2\omega_xt+\theta_x)\\&\quad-\frac{k\omega_x}{\omega_y(\omega_x+\omega_y)}\left(1+\frac{k^2}{4\omega_y^2}\right)^{1/2}\sin(2\omega_yt+\theta_y)\,,\end{aligned} \tag{43}$$

where

$$\theta_x=\cos^{-1}\left(1+\frac{\mu^2}{4\omega_x^2}\right)^{-1/2},\qquad \theta_y=\cos^{-1}\left(1+\frac{k^2}{4\omega_y^2}\right)^{-1/2}. \tag{44}$$

From the above expression, we see that the maximum and minimum of K_i^2 depend on the ratio of ω_x/ω_y as well as the values of θ_x and θ_y, while the upper and lower limits of K_i^2 are

$$1+\frac{\mu^2\omega_y}{2\omega_x^2(\omega_x+\omega_y)}+\frac{k^2\omega_x}{2\omega_y^2(\omega_x+\omega_y)}\pm\left[\frac{\mu\omega_y}{\omega_x(\omega_x+\omega_y)}\left(1+\frac{\mu^2}{4\omega_x^2}\right)^{1/2}+\frac{k\omega_x}{\omega_y(\omega_x+\omega_y)}\left(1+\frac{k^2}{4\omega_y^2}\right)^{1/2}\right]>0\,. \tag{45}$$

The plot of the dependence of the minimum values of K_i^2 on the parameters of delta-kicks k and λ is shown in Fig. 1. The frequency ω_0 is taken to be equal to unity. When the parameters of delta-kicks are equal to zero the minimum of the squeezing coefficient is equal to unity, i.e., there is no squeezing phenomenon in this case. Then

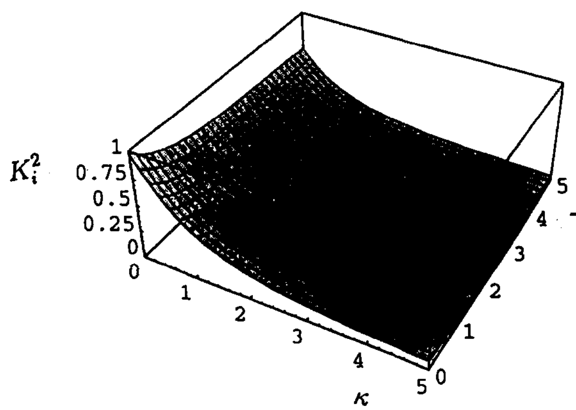

Fig. 1. The dependence of the squeezing coefficient K_i^2 versus the parameters of delta-kicks k and λ.

the minimum of the squeezing coefficient is decreasing while the parameters of delta-kicks are increasing. So the stronger kicks correspond to the stronger squeezing phenomenon. As can be seen from the plot in Fig. 1 the squeezing is more sensitive to the parameter k of the kicks in the oscillatory frequency. The kicks of the interaction frequency determined by the parameter λ produce the squeezing also but the dependence of the squeezing on the parameter λ is less then on the parameter k. For example, in the case $k=0$, $\lambda=5$, one has $K_i^2=0.6...$, but for $k=5$, $\lambda=0$, one basis the value of the minimum of the squeezing coefficient $K_i^2=0.06...$ which is ten times smaller.

If the delta-kicks are weak, i.e., $k, \lambda \ll \omega_0$, then

$$K_i^2(t>0)=1-\frac{1}{\omega_x+\omega_y}\left(\frac{\mu\omega_y\sin(2\omega_x t)}{\omega_x}+\frac{k\omega_x\sin(2\omega_y t)}{\omega_y}\right), \tag{46}$$

and the upper and lower limits of K_i^2 become

$$1\pm\frac{k\omega_x^2+\mu\omega_y^2}{\omega_x\omega_y(\omega_x+\omega_y)}. \tag{47}$$

Analogous to K_i, the correlation coefficients Γ_{ij} $(i=1,2)$ are defined as

$$\Gamma_{ij}=\frac{\sigma_{q_ip_j}}{\sqrt{\sigma_{q_iq_i}\sigma_{p_jp_j}}}, \tag{48}$$

which describe the statistical dependence of the operators of coordinates and momenta in this state. If the initial state is a coherent state, then $\Gamma_{ij}=0$ before the kicks and become nonzero after the kicks. Hence we conclude that the delta-kicks excite the coherent state into "correlated" squeezed states.

4. Application

The system we considered in this paper may model the interacting resonant circuits [16] and Josephson junction [17] with rapidly changing parameters. For example, consider the Hamiltonian

$$\hat{H}=\tfrac{1}{2}L_1\hat{I}_1^2+\tfrac{1}{2}L_2\hat{I}_2^2+\frac{\hat{Q}_1^2}{2C_1}+\frac{\hat{Q}_2^2}{2C_2}+L_{12}(t)\hat{I}_1\hat{I}_2, \tag{49}$$

where L_1 and L_2 are the inductances, C_1 and C_2 are the capacitances and $\hat{I}_1$ and $\hat{I}_2$ are the operators of the currents of the first and second circuit, respectively. The rapid change of the self-inductance L_{12} can be modeled either by a single delta-kick

$$L_{12}=L_0+k\delta(t-t_0), \tag{50}$$

or by a series of delta-kicks

$$L_{12}=L_0+\sum_{n=1}^{N}k_n\delta(t-t_n), \tag{51}$$

where L_0 is the constant part of the self-inductance.

If the circuit is made of two integrating Josephson junctions, then (49) becomes

$$\hat{H}=\frac{I_{c_1}\hat{\phi}_1^2}{4e}+\frac{I_{c_2}\hat{\phi}_2^2}{4e}+\frac{\hat{Q}_1^2}{2C_1}+\frac{\hat{Q}_2^2}{2C_2}+L_{12}I_{c_1}I_{c_2}\hat{\phi}_1\hat{\phi}_2, \tag{52}$$

where $\hat{\phi}_1$ and $\hat{\phi}_2$ are the phase operators, I_{c_1} and I_{c_2} are the critical currents, and e is the electron charge. This model is valid in the case of small phases with [17]

$$I_{c1}C_1, I_{c2}C_2 \geqslant 32e^3 \approx 10^{-21} AF. \tag{53}$$

One can parametrically excite the Josephson junctions by varying their plasma frequencies or the parameter of the self-inductance. This can be accomplished by a specified time variation of capacitances or critical currents. Since the critical currents are due to the tunnel effect, i.e., depend exponentially on the thickness of the junction or on certain other parameters, it is much easier in practice to obtain noticeable time variation of critical currents than capacitances. In addition, it was shown in Ref. [18] that it is possible to obtain a manifold of the junction current by infrared irradiation or by ultrasound irradiation. In Ref. [19] the feasibility of using a Josephson junction to generate squeezed electromagnetic radiation was considered. In Refs. [17,20,21] the possibility of generating the correlated squeezed states in the Josephson junction by parametric excitation was suggested. Applying our results one can calculate the covariance matrix and other parameters of the correlated squeezed states generated in these circuits.

5. Discussion

In this paper we have only considered the ideal case and ignored the dissipation. In order to take the dissipative mechanism into account, we have to either (i) use the quantum Focker–Planck equation instead of the Liouville equation for the evolution of the Wigner functiion [22], or (ii) couple the system to a dissipative heat bath [23]. This generalization will be discussed in another paper.

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